FIRST PLACE IN THE POSTER CONTEST "CARLOS GRAEF FERNÁNDEZ" DEL LXII CONGRESO NACIONAL DE FÍSICA 2019

BLACK HOLE DISPERSION: HAYWARD

—

Pedro Josué Hernández Penagos[1]
pedrojosuehernandez01@gmail.com

Omar Pedraza Ortega[2]
omarp@uaeh.edu.mx

Roberto Arceo Reyes[1]
roberto.arceo@unach.mx

1 Faculty of Science in Physics and Mathematics, Universidad Autonoma de Chiapas, México

2 Academic Area of Mathematics and Physics, Universidad Autonoma del Estado de Hidalgo, México

Para citar este artículo:
Fraga R. Evelyn L. (2020). [Reseña del libro Planificación para el desarrollo en América Latina y el Caribe. Enfoque, experiencias y perspectivas, de Máttar, J., & Cuervo, L. M.]. *Espacio I+D, Innovación más Desarrollo. IX*(22), 178-185. Recuperado de https://espacioimasd.unach.mx/docs/Dispersion_de_Hoyos_Negros_Hayward.php

RESUMEN

The current work analyses flat wave scattering problems that impact on a black hole. In this case, a Hayward black hole is worked on, using the Klein-Gordon equation for a massless scalar field, and the differential equation obtained is solved numerically to find the differential effective section.

Black holes are objects so compact that not even light can escape their gravitational pull. Nowadays, existence of supermassive black holes (millions of times the mass of the sun) in the center of many galaxies, and smaller black holes (5-10 times the mass of the sun) in binary X-ray systems is generally accepted [1].

The Schwarzchild black hole, the simplest, has spherical symmetry and is described by the line element

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{1}{\left(1 - \frac{2M}{r}\right)} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\varphi^2, \qquad (1)$$

here, M is the mass of the black hole (in natural units).

In this paper, we will study scattering problems involving black holes. This problem is similar to a scattering problem, where the refractive index of the medium is not constant. In black holes case, curvature of space-time produces the scattering effect of the incident waves.

Let us consider a plane wave $\boldsymbol{\psi}(z)=\mathbf{e}^{i\omega z}$ that strikes the black hole. Information from the black hole is encoded in an effective long-range potential. Effective potential on the incident plane wave modifies standard expressions (plane space) for the scattering amplitude.

The scattered wave (see Image 1) can be expressed as

$$\psi(r,\theta) \approx Ae^{i\omega z} + f(\theta)\frac{e^{i\omega r}}{r};$$

$$f(\theta) = \frac{1}{2i\omega}\sum_{l=0}^{\infty}(2l+1)(S_l - 1)P_l(\cos\theta), \qquad (2)$$

where $\mathbf{f}(\theta)$ is the amplitude of dispersion, S_l is the S matrix and $P_l(\cos\theta)$ are the Legendre polynomials of order l.

From the scattering amplitude, the differential effective cross section [2] is obtained, given by

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2. \qquad (3)$$

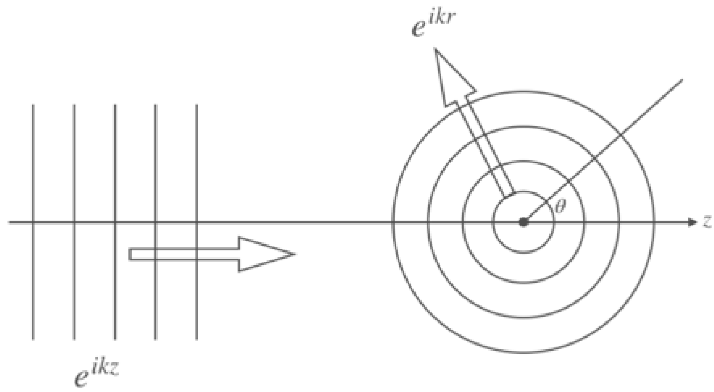

Image 1. Incident plane wave traveling in the z-axis and the scattered spherical wave

SCALAR FIELD EQUATION AND EFFECTIVE POTENTIAL

Reference [3] describes a regular black hole, this model has spherical symmetry, is asymptotically flat and has no singularities in $r = 0$.

Space-time of the Hayward regular black hole line element is

$$ds^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2 d\Omega^2, \quad F(r) = 1 - \frac{2Mr^2}{r^3 + 2M\epsilon^2}, \tag{4}$$

where M represents the mass of the black hole, the parameter $\boldsymbol{\varepsilon}$ is associated with a cosmological constant. For $\boldsymbol{\varepsilon} = 0$, the Hayward metric is reduced to the Schwarzschild metric.

Regular Hayward black hole describes the behavior of a collapsing or evaporating black hole and may have one or two event horizons, depending on the relationship between mass and parameter $\boldsymbol{\varepsilon}$.

Consider a massless scalar field $\boldsymbol{\psi}$ that propagates in Hayward space-time. The equation governing the evolution of the scalar field is

$$\frac{1}{\sqrt{g}} \partial_\mu \left(\sqrt{g} g^{\mu\nu} \partial_\mu \psi \right) = 0. \tag{5}$$

For monochromatic plane waves, we have

$$\psi_\omega = \sum_{l,m} \frac{\phi_l(r)}{r} Y_l^m(\theta, \varphi) e^{-i\omega t}. \tag{6}$$

with $\psi_\omega = Y_l^m(\boldsymbol{\theta}, \boldsymbol{\varphi})$ the spherical harmonics. Substituting (6) for (5), the following equation is obtained

$$F(r)\frac{d}{dr}\left[F(r)\frac{d}{dr}R_l(r)\right]+[\omega^2-V(r)]R_l(r)=0\,, \tag{7}$$

where we have considered that **φ=0**. The effective potential *V(r)* is given by

$$V(r)=F(r)\left[\frac{1}{r}\frac{dF(r)}{dr}+\frac{l(l+1)}{r^2}\right]. \tag{8}$$

The effective potential is plotted in Image 2.

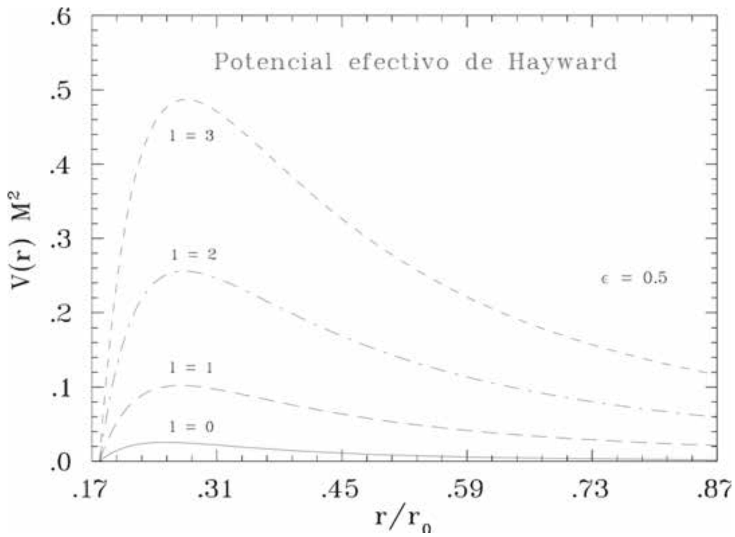

Image 2. Effective potential (8) for a scalar wave in a Hayward black hole, with *l = 0, 1, 2, 3* for *M* = 1 and ε = 0.5

NUMERICAL SOLUTION

Solution to equation (7) is possible by using a numerical method, defining the function's values in discrete quantities on a uniform nodal spacing grid. The differential equation becomes a finite difference equation and can be solved along a grid by calculating new values of the function from previously known values.

Using Taylor's second order expansion for a *u(r)* function. Solving the first and second derivatives of *u(r)* (equations (3.5) and (3.26) of [4]) gives

$$u_i' \approx \frac{u_{i+1}-u_{i-1}}{2h}; \qquad u_i'' \approx \frac{u_{i+1}-2u_i+u_{i-1}}{h^2}; \tag{9}$$

where h is the interval between the nodes and the subscript in u label the nodal point. Substituting in equation (7) and clearing *ui+1*

$$u_{i+1} = \frac{2F(r)h^2}{2F(r)+\frac{dF(r)}{dr}}\left[\frac{2u_i - u_{i-1}}{h^2} + \frac{\frac{dF(r)}{dr}}{F(r)}\frac{u_{i-1}}{2h} + \frac{1}{F(r)}[\omega^2 - V(r)]u_i\right] \quad (10)$$

Amplitude of dispersion $f(\boldsymbol{\theta})$ can be rewritten as follows [4]

$$f(\theta) = \frac{1}{2i\omega}\sum_{l=0}^{\infty}(2l+1)(S_l - 1)P_l(\cos\theta) \quad (11)$$

where S_l is called the **S-matrix**. Which is written as

$$S_l = \frac{u_l(r_{n-1})r_n h_l^{(-)}(\omega r_n) - u_l(r_n)r_{n-1}h_l^{(-)}(\omega r_{n-1})}{u_l(r_n)r_{n-1}h_l^{(+)}(\omega r_{n-1}) - u_l(r_{n-1})r_n h_l^{(+)}(\omega r_n)} \quad (12)$$

here $\mathbf{h}_l^{(\pm)}$ (ωr_n) are the spherical Hankel functions and u_l (r_n) sare the solutions of equation (10).

RESULTS

The following figure shows the numerically calculated differential effective dispersion section and the Glory approximation.

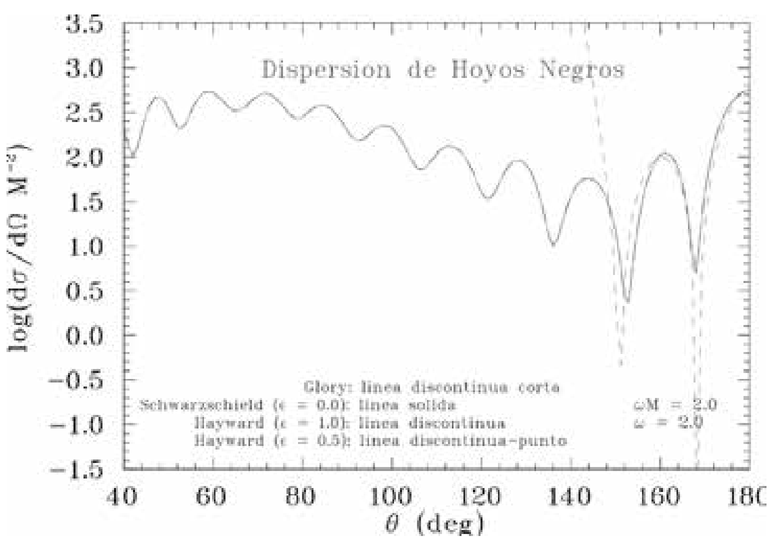

Image 3. Differential RMS section for Hayward metric

CONCLUSIONS AND STANDPOINT

- Results obtained coincide with the Glory dispersion formula.
- Analyze black hole scattering problem considering electromagnetic and gravitational fields.
- Other black hole scenarios can also be analyzed.

REFERENCES

[1] **N. Andersson and B. P. Jensen**, (2001). arXiv:gr-qc/0011025

[2] **D. J., Griffiths**, (2005), *Introduction to Quantum Mechanics*, USA, Pearson Education, Inc.

[3] **S. A. Hayward**,(2005), arXiv:gr-qc/0506126.

[4] **Gibbs, W. R**. *Computation in Modern Physics*; 3rd Ed., Singapore, MI, USA, 2006.

ANNEX


La Sociedad Mexicana de Física

Otorga el presente

RECONOCIMIENTO

A

Roberto Arceo Reyes, Universidad Autónoma de Chiapas;
Pedro Josué Hernández Penagos, Universidad Autónoma de Chiapas;
Omar Pedraza Ortega, Universidad Autónoma del Estado de Hidalgo

Por ser uno de los ganadores en el Concurso de Carteles
"CARLOS GRAEF FERNÁNDEZ",
M8-522, "Dispersión de hoyos negros: Hayward"
8 de octubre de 2019

LXII Congreso Nacional de Física 2019

Dr. Tonatiuh Matos Chassin
Presidente de la
Sociedad Mexicana de Física