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Mechanics Problems for Olympiad

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These notes contain a brief collection of Newtonian classical mechanics problems intended for the Physics Olympiad training of the 2020 Mexican national team that represented Mexico in the international Olympiad, adding to similar collections that already exist. Some problems were compiled from classic sources such as well-known worksheets or other Olympiads, others I have modified or invented to form a collection that includes original problems in the spirit of the (now defunct) Editorial Mir and the style of the "old school" Physics Olympiad problems. The answers to the problems are given at the end of the text with different degrees of detail in the resolution, while in some cases complete solutions are given, in others only the answer is shown. This is intentional and its purpose is to encourage the reader to find their own and original solutions to the proposed problems that are presented as well as a challenge in the learning process. Following the tradition of Olympiad problems, elemental, but not simple, problems have been proposed. This short collection of problems could also be useful as supplementary material in higher-level classical mechanics courses.

I want to thank Dalí Pinto and Jairo Villalobos, students of the Physics degree at the Universidad Autónoma de Chiapas, for helping me elaborate the figures and diagrams, as well as for endless discussions of physics and mathematics problems. Although the manuscript has been revised on several occasions, any errors that remain in it are my responsibility, and for this reason, I appeal to readers that if they find errors, they let me know at idrish.huet@gmail.com so that they could be corrected at a later version.

I hope these problems will be of interest to Olympiad students and as interesting and useful at this stage in their learning of physics as other similar collections of problems were for me at the time.

Idrish Huet Hernández
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CLASSICAL MECHANICS PROBLEMS

1. There are five ants on the vertices of a regular pentagon of a, side, each one moves at a u speed always following its neighbor to the right (Fig. 1). Determine how long τ take for them to meet.

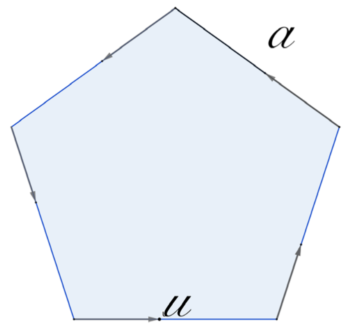


Figure 1

2. A solid and homogeneous sphere with an R radius falls under its own weight down a ladder with square steps of side $a \ll R$, the sphere never loses contact with the ladder (Fig. 2). Find the terminal velocity of the center of the sphere v_∞ when (a) the sphere has no friction with the ladder (b) the sphere has so much friction with the ladder that it rolls without slipping as it falls down the ladder.

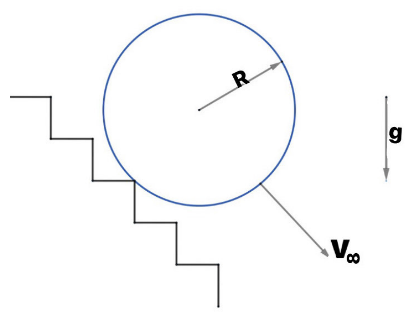


Figure 2

3. An inextensible thread of length L attaches a nail of radius $r \ll L$ to a mass m. The mass moves on a frictionless horizontal table where the nail is fixed. Initially, with the thread fully extended, velocity v_0 is communicated to the mass (Fig. 3). According to the principle of conservation of angular momentum, we should have that $mv_0L = mv\ell$ so that $v = v_0L/\ell$ will be

the speed of the mass when the length of the thread is ℓ. However, as the thread is inextensible, it cannot do work on the mass because its tension is always per-linked to the movement of the mass, so the velocity cannot change. Solve this paradox and find the time t qit takes to wrap the string to length $\ell = L/2$.

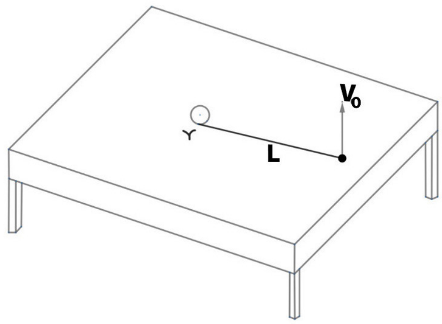


Figure 3

4. A cube collides elastically with a wall so that its velocity makes an angle α con la misma. with the wall. The coefficient of friction of the wall with the cube is μ. Find the angle β that the velocity of the cube makes with the wall after the bounce (Fig. 4).

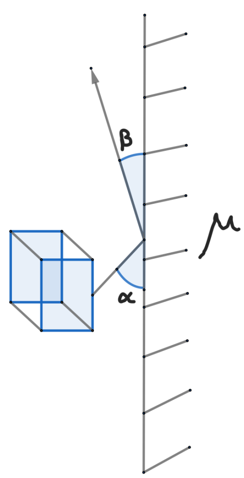


Figure 4

5. A thin spherical shell of weight W rests on two legs so that the points of contact are separated by the angle α (Fig. 5). (a) Calculate the pressure force on each leg. One of the legs is suddenly withdrawn, calculate the pressure force on the other leg an instant later if (b) The legs are so rough that they

do not allow the sphere to slide. (c) The sphere slides without friction on the material of the legs.

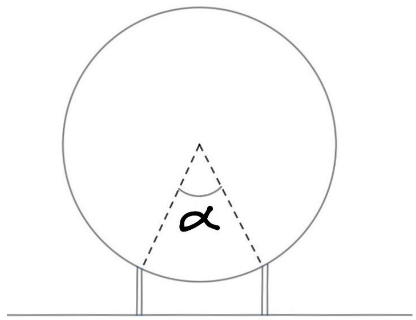


Figure 5

6. A chain of uniform weight W and length L hangs vertically above a table so that the lower end just grazes the surface. (a) The chain is allowed to free-fall, what is the maximum force exerted by the chain on the table? Now consider a different situation: The chain is resting on the table and one end is started to be pulled to lift it, with the force f at a $t = 0$. time. (b) How must $f(t)$ depend on the time for the end of the chain rises with constant acceleration $g/2$? (c) What should $f(t)$ of time for the end to rise with constant speed u? (d) Considering the three previous situations, what is the tension at the midpoint of the chain when $2/3$ of its length are in the air?

7. A hollow spherical shell of radius R is filled with a liquid of density ρ. The sphere and the liquid rotate with angular speed ω on a vertical axis passing through the center of the shell (Fig. 6) (a) Find the pressure $P(\theta)$ on the inner surface of the shell. (b) Find the value of the maximum pressure P_{max} and the angle θ_0 where it occurs.

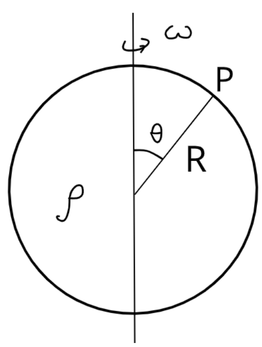


Figure 6

8. A projectile is launched vertically until it reaches the height $3R$ above the earth's surface, being $R \approx 6370\,km$ the radius of the earth. The projectile is launched at a latitude such that when it lands it falls very close to where it was launched. How long does it take to land once launched? The formula for the segment's area (shaded) of the ellipse (Fig. 7) is

$$A(h) = ab\arccos(h/a) - bh\sqrt{a^2 - h^2}/a$$

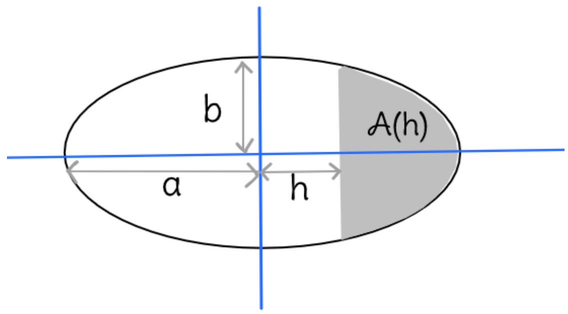


Figure 7

9. A marble is thrown with horizontal velocity against an inclined plane which bounces elastically exactly n times, the first and last (n-th) collisions occur at the same point (Fig. 8). After the last collision, the marble's speed has the opposite direction to its initial speed. Find α, the angle of inclination of the plane concerning the horizontal in terms of n.

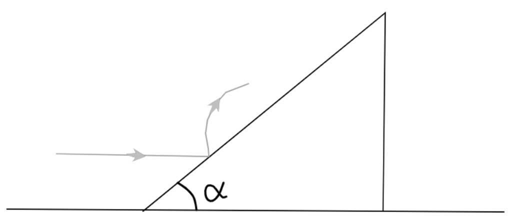


Figure 8

10. A block of mass M can slide without friction on a table. Against the block rests a homogeneous bar of mass m that has the same height L as the block and can rotate freely using a pivot that joins it to the table. Initially, the bar is vertical and completely glued to the block, after a small initial push the bar slides slowly pushing the block (Fig. 9) What value must M/m have

so that the angle ϕ with the vertical is $\frac{\pi}{3}$ at the time the block and the bar separate? (b) Calculate the velocity of the block when this occurs.

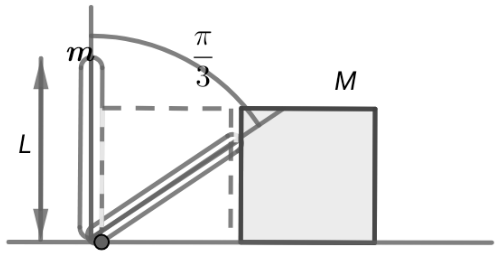
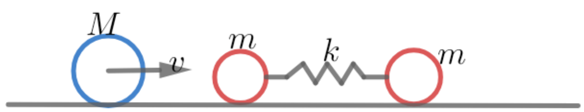


Figure 9

11. A particle with charge q and mass m starts from rest at a distance z z from a metal plate that can be considered infinite. Find the time τ it takes to reach the plate.

12. Two masses m eare linked by a spring of elastic constant k and rest on a frictionless table. Suppose that all movement of the masses occurs on a line. A mass M collides elastically with one of the masses m (Fig. 10). Find the values of $\xi = \frac{m}{M}$ for which a second collision occurs between the



masses m and M.

Figure 10

13. A planet is in a liquid state and the density of this liquid (magma) can be constant and equal ρ. In the process of planet formation, the magma rotates uniformly with angular speed ω and approximately acquires the shape of an ellipsoid of revolution with semi-axes $a > b$ (Fig. 11). The flattening factor is defined as $f = \frac{a-b}{a}$ and satisfies $f \ll 1$. Calculate f for Earth, using $\rho = 5.5 \times 10^3\, kg/m^3$ nd compare the result with the actual value $f_T = 3.35 \times 10^{-3}$. For simplicity, assume that the gravitational force can be

calculated for this ellipsoid by conceiving all the mass as concentrated at the center of mass.

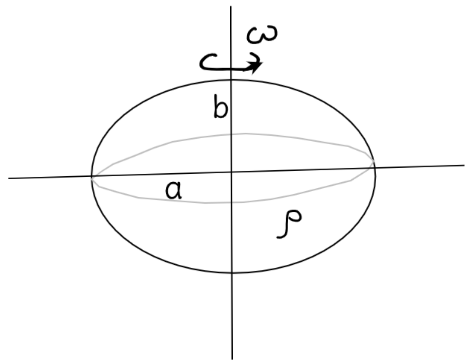


Figure 11

14. An object slides down a smooth curved ramp from point A to B dwhere its velocity makes the angle $\pi/4$ with the horizontal, as shown in the figure. Initially, a horizontal speed is communicated to it starting from A ynd it is observed that during its movement the speed remains constant. Point A is at the height h and horizontal distance ℓ from B. Upon reaching B the object bounces elastically against the ground so that it falls to a distance ℓ from BB at point C (Fig. 12). Calculate the coefficient of friction μ between the ramp and the object, assuming the object does not lose contact with the ramp between A and B. Assume further that the curvature mean radius of the ramp is comparable to h.

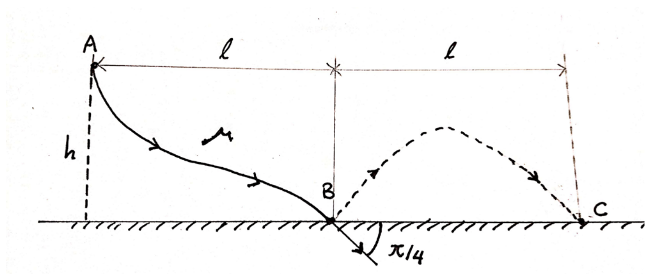


Figure 12

15. How far from the center must a hollow sphere be struck horizontally for it to begin rolling without slip when it is initially at rest on a very slippery ice surface?

16. You have a very fine chain of length L with a large number N of links on a frictionless table. The chain can be considered homogeneous. Initially, the chain is at rest with one of the links hanging vertically over the corner of the table (Fig. 13). The chain begins to slide. Find how long τ it takes for the chain to fall completely off the table.

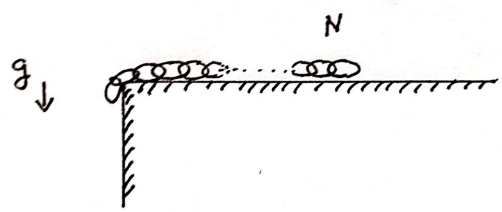


Figure 13

17. Many spheres slide without friction along a channel that starts from a fixed point O but can be tilted at any angle concerning the horizontal so that the spheres fall by gravity. If many spheres are released simultaneously from O through many channels covering many angles, the spheres always lie on a curve that changes dynamically in time (Fig. 14). (a) 14). (a) Find this curve when the spheres start from rest (b) Find this curve when each sphere has been given an initial velocity u.

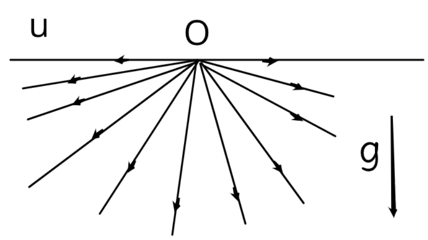


Figure 14

18. A sphere collides elastically with a wall in such a way that the angle between its velocity v and the wall just before the collision is α. The wall moves towards the sphere with constant velocity u (Fig. 15). (a) (a) Find the value of u necessary so that the sphere bounce angle is 2α. (b) For which α values is this possible?

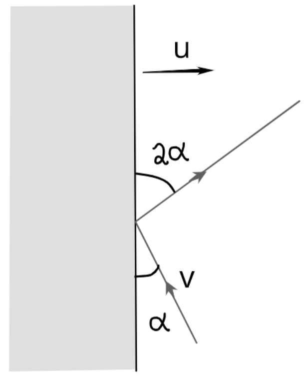


Figure 15

19. A ring of radius r and mass m is lying on a table, see figure. The ring rotates with angular speed ω and slides on the horizontal surface of the table that has a coefficient of friction μ with the ring. Initially, the velocity v_0 is communicated to the ring (Fig. 16). (a) Describe the path of the ring. (b) Explain in which case, and why, the ring travels a greater distance, when it rotates or when it does not rotate.

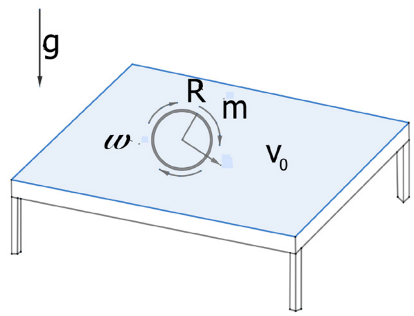


Figure 16

20. Consider a wire in the shape of S (Fig. 17) that has two elbows that gently bend at an angle $\pi/2$ each. A bead slides along the wire with initial velocity v_0. Find the final velocity of the bead if the coefficient of friction between it and the wire is μ. The movement occurs in a horizontal plane, so gravity does not play a role.

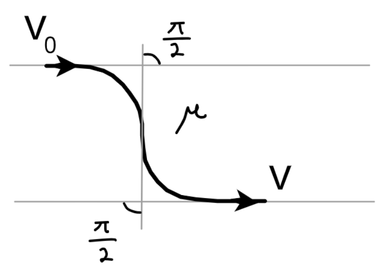


Figure 17

21. A beam of light is fired from the center of a disk in a radial direction. The disc rotates with speed ω (Fig. 18). An observer rotating with the disk will determine that the light does not move in a straight line, what path will the light follow for this observer?

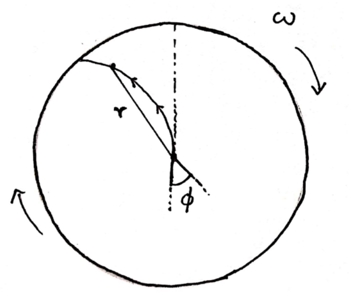


Figure 18

22. You have a cylindrical container with a small hole in the bottom, you put liquid in it and stir it a little so that it forms a swirl as it drains. Consider that the swirl is axially symmetric and make the necessary approximations (Fig. 19). (a) Find the profile (shape) of the surface of the liquid in the container. (b) If the liquid in the contained swirl has the volume V, find the swirl's height h at the edge of the container if its radius is a and the hole has radius b.

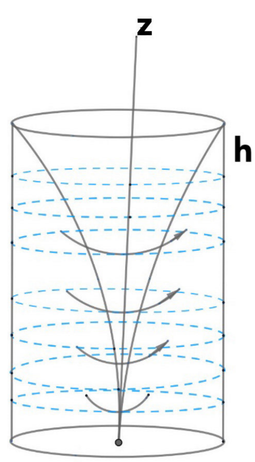


Figure 19

23. A mass M is suspended vertically from an elastic with Hooke's k constant and mass m. An approximation (not so bad in this case) is to consider that the elastic's linear density is always uniform. Calculate the frequency of the oscillations of the mass M.

24. A comet orbiting the Sun has impact parameter b and velocity at infinity v_0 (Fig. 20). Calculate (a) the comet's maximum velocity v_{max} (b) its minimum distance r_{min} to the Sun. Take the mass of the Sun as M.

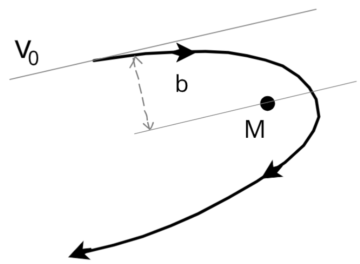


Figure 20

25. A homogeneous rope of weight W hangs from the ceiling from its two ends at points A and B very close together as shown in the figure. The rope is attached to the ceiling through a ring at A, which breaks when subjected to tension greater than or equal to F. At one point the rope is released from the B end while the A end remains attached (Fig. 21). Find the minimum necessary value of F so that the ring does not break.

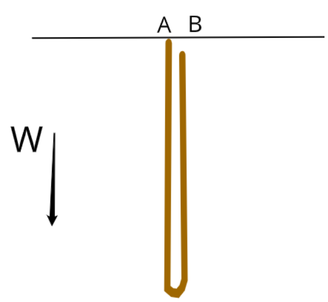


Figura 21

26. You are in an empty room, and you only have at your disposal a blackboard, chalk, and a measuring tape. Explain how to calculate the distance between your pupils.

27. Two soap bubbles with $r_1 > r_2$ radius are glued (Fig. 22). (a) Find the radius of curvature ρ of the surface that forms the interface between them. (b) now consider that $r_1 = r_2 = r$, the stuck bubbles suddenly merge into a single bubble. Find the radius R of this new bubble. Assume that the pressure due to the surface tension of the bubbles is much less than the

atmospheric pressure so it is not necessary to consider changes in the volume of the air within the bubbles. Use the formula for the volume of a spherical dome:

$$V(r,h)=\frac{\pi}{3}(2r^3-3r^2h+h^3)$$

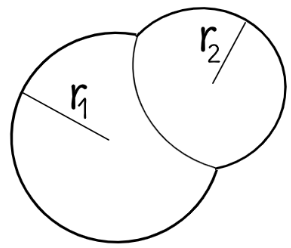


Figure 22

28. A cycloid is generated by a point on a circle of radius R in the plane xy that rolls without slipping on the axis x. The point touches the axis x at A and reaches its highest point B opposite to E (Fig. 23). Calculate the radius of curvature ρ of the cycloid at the point D of the cycloid whose coordinate x is at the point C that bisects AE (it is possible to do it by kinematics).

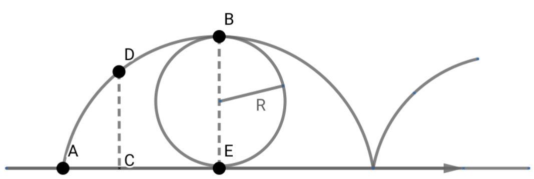


Figure 23

29. In a similar way to the previous problem, consider an epicycloid, the curve generated by a fixed point of a circle of radius r that rolls without slipping on a circle of radius R and center O. Calculate (it is possible to do it by kinematics) the radius of curvature ρ at point Q, the furthest to O from the epicycloid (Fig. 24).

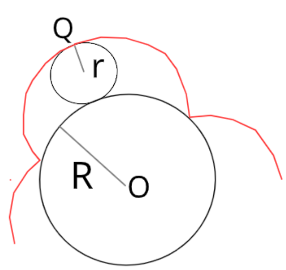


Figure 24

30. A homogeneous and thin wand of length L has a pivot in the center and can rotate freely without friction. Initially, the wand is in equilibrium in a horizontal position when a spider lands $t = 0$ at the midpoint between the center and one end with vertical velocity v_0 (Fig. 25). Upon landing the spider begins to run towards the nearest shore in such a way that the angular speed ω_0 of the wand remains constant. The mass of the spider is half the mass of the wand. (a) Find ω_0 (b) Find the $u(t)$ speed the spider must run for the angular speed to be constant. (c) Find v_0 so that when the spider reaches the end of the wand it is upright.

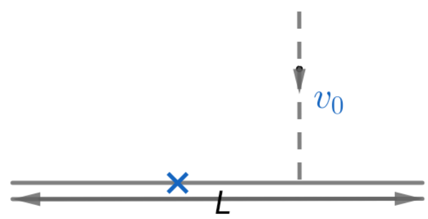


Figure 25

31. A particle is launched from the ground in a parabolic throw so that it skims the upper surface of a sphere of radius R resting on the ground (Fig. 26). (a) Find the minimum launch speed u (b) Find the corresponding launch angle θ to the horizontal.

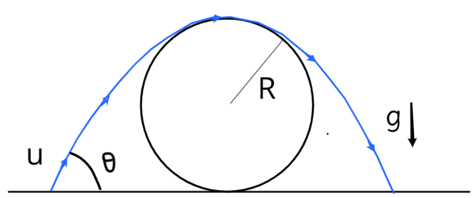


Figure 26

32. From the base of an inclined plane at an angle β to the horizon a small sphere is thrown in a parabolic shot. Find the launch angle α, concerning the horizontal, so that the sphere hits the point of the inclined plane as far as possible from where it started (Fig. 27).

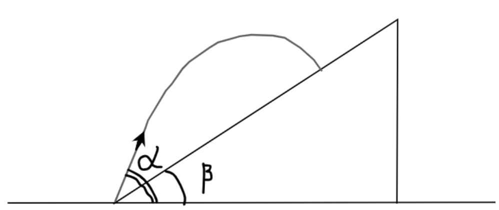


Figure 27

33. A satellite of mass m is in the atmosphere where it is subjected to a force of air resistance $\vec{f} = -\alpha\vec{v}$, the air resistance slows it down so that it will end up crashing to Earth. Originally, it orbits at a height above the surface equal to half the Earth's radius and the fall of the satellite occurs slowly. How many turns will the satellite make before impacting the earth?

34. A cylinder of radius R that rolls without slipping hits an edge that is at the height h (Fig. 28). Find the maximum value of h for the cylinder to still clear the rim when (a) during the collision there is no friction between the cylinder and the rim. (b) during the collision, there is no slip between the cylinder and the rim. In both cases, once the cylinder contacts the edge, consider that during its ascent it does not lose contact.

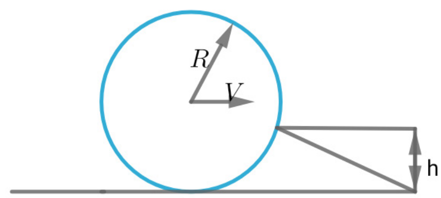


Figure 28

35. A fountain F moves at velocity v while emitting sound with wavelength λ, where u is the speed of sound and $v < u$. The distance from the source F to the receiver A is L and the angle between FA and the velocity of F is ϕ (Fig. 29). Find the apparent frequency f' that A receives.

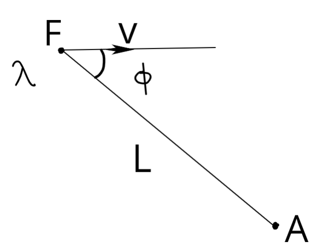


Figure 29

36. A ship is tied to a mooring by a rope that is wrapped around a bollard (low-height post) at an ϕ angle. The free end of the rope is pulled by a sailor who can exert a maximum force f. The coefficient of friction between the

rope and the bollard is μ (Fig. 30). What is the maximum force F with which the ship could be pulled so that the sailor can still stop it with the rope?

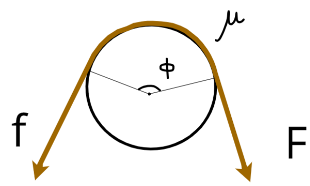


Figure 30

37. There is a thin-walled pyramid-shaped container with no base that rests on a table. At the top of the pyramid, there is a small hole through which water is slowly poured (Fig. 31). By the time the container is filled with water, it begins to escape from the bottom of the container. When the container is full of water, the weight of the water it contains is W. Calculate the weight W_0 of the container.

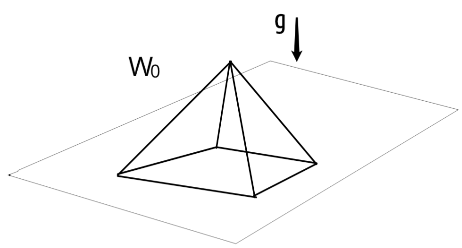


Figure 31

38. Two rafts drift on a lake, initially start from the same point, and begin to move apart so that their speeds make an angle of $\pi/3$. The speed of one raft is always twice the speed of the other, both speeds are constant in magnitude and direction. On each raft, there is a clock with hands where the second-hand measures $\ell = 10\ cm$, the clocks are identical. At each instant of time, it is always possible to orient each watch so that the tips of the second hands are at relative rest (Fig. 32). (a) What is the speed of each raft? (b) Find the distance between the rafts after two days.

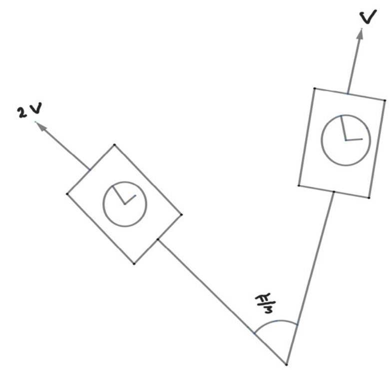


Figure 32

39. At which point $\boldsymbol{P}$ of a long and homogeneous rod should we hit so that if we take it from one end, we do not feel the rebound? (Fig. 33)

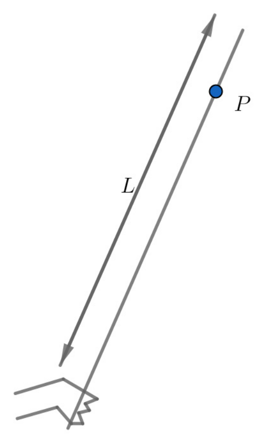


Figure 33

40. A bead slides from a frictionless point $\boldsymbol{Q}$ down a wire that is inclined at an angle $\boldsymbol{\beta}$ to the vertical and comes down the wire to some point $\boldsymbol{P}$ on a plane inclined at an angle $\boldsymbol{\alpha}$ to the horizontal. What angle $\boldsymbol{\beta}$ should be chosen so that the $\boldsymbol{Q}$'s time to plane is minimal? (Fig. 34) Consider that the wire is always long enough to go from $\boldsymbol{Q}$ to touch the plane.

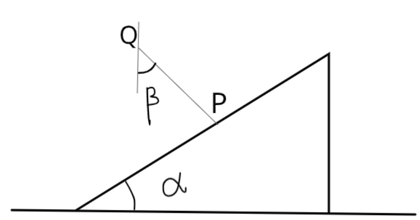


Figure 34

41. A force F is applied horizontally to the center of a hollow cylinder of weight W and radius R which rests on the ground against a step height $h < R$ (Fig. 35). What is the minimum value of F such that the cylinder will clear the step?

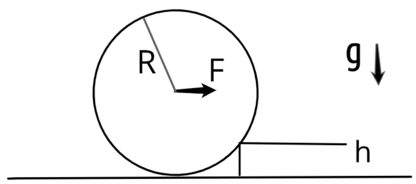


Figure 35

42. Identical n beads slide without friction down a vertical wire and fall under their weight. Each bead is initially given a velocity v_i, $i = 1,2,\cdots,n$ which can be directed up or down the wire, the velocities v_i can, at first, all be different in magnitude. The collisions between the beads are fully elastic (Fig. 36). (a) Calculate the maximum number of collisions N that is possible between the beads (b) If the average initial velocity of the beads is v equal to time $t = 0$ find the time required t for the kinetic energy of the beads to return to the initial value. (c) If the n beads initially all have downward velocities u_i and are equally spaced the distance d. Find values for each u_i such that all beads are found at the same time t.

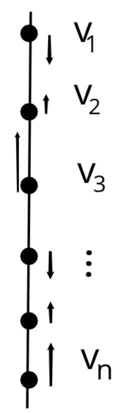


Figure 36

SOLUTIONS

1. Ants are always found at the vertex of a regular pentagon. Going to the rest system of some of them, it is found that the speed with which the edge between two neighbors decreases is constant and equal to $u\left(1-\cos(2\pi/5)\right)$. From this, we have $\tau = \frac{a}{u}\left(1+\frac{1}{\sqrt{5}}\right)$. Another way is to decompose the movement of each ant instantaneously into a rotation concerning the center of the pentagon and scaling of the pentagon (radial speed), using this method it is also possible to show that the ants move following a logarithmic spiral.

2. In both cases, the trajectory of the mass' center is the same. The sphere can never have contact with more than two points, but always with at least one. In both cases, the conservation of angular momentum concerning the point of each impact allows us to find the relationship between the velocities of the mass' center before and after each impact. (a) In this case, the sphere never rotates when it falls, the law of conservation of energy tells us $\frac{1}{2}mv_\infty^2(1-\cos^2\varphi) = mga$. As a result, $v_\infty = R\sqrt{g/a}$ (b) In this case the sphere rotates as it falls and the kinetic energy of rotation must be considered if $v_\infty = \omega_i R$ we have $\frac{1}{2}I\left(\omega_i^2 - \omega_f^2\right) = mga$, where $I = \frac{7}{5}mR^2$. Conservation of angular momentum $I\omega_f \approx I\omega_i - m\omega_i a^2$ implies hence $v_\infty = R\sqrt{g/a}$. In case (b) the friction forces do not work, so the result is the same for v_∞.

3. Conservation of angular momentum cannot be applied to the nail even though its radius is very small; the point of contact of the thread with the nail rotates. The binding forces do not work, so the velocity of the dough is constant. The movement can be considered as a series of instantaneous circular movements concerning the spin radius L that varies with time. As can be seen in the figure $dL = -rd\varphi$, and at each instant$v_0 dt = Ld\varphi$, it results in $\tau = \int_L^{L/2} \frac{dt}{dL}\,dL = \frac{3L^2}{8rv_0}$.

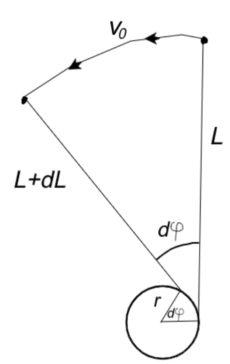


Figure 37

4. The tangential moment change to the wall is $-\mu$ times the normal moment change to the wall. $\beta = \operatorname{arccot}(\cot\alpha - 2\mu)$ if $\mu \leq \cot\alpha/2$, $\beta = \pi/2$ if $\mu > \cot\alpha/2$.

5. The static equilibrium condition implies (a) (a) $N = \frac{W}{2}\sec(\alpha/2)$. (b) Here we must use the rigid body equations of motion. The angular acceleration is $\dot{\omega} = \frac{3g}{5R}\operatorname{sen}(\alpha/2)$, and the friction force $f = \frac{2}{5}W\operatorname{sen}(\alpha/2)$, from this we have $N = W\cos(\alpha/2)$. (c) In this case, the acceleration of the center of mass of the sphere is $a = g\operatorname{sen}(\alpha/2)$, so $N = W\cos(\alpha/2)$.

6. (a) $F_{max} = 3W$. (b) $f(t) = \frac{3gt^2}{8L}W$. for $0 \leq t \leq 2\sqrt{L/g}$, $f(t) = \frac{3}{2}W$ for $t > 2\sqrt{L/g}$. (c) $f(t) = W\frac{ut}{L}$ for $0 \leq t \leq L/u$, $f(t) = W$ for $t > L/g$. (d) The tension in each case is: $T_{(a)} = 0$, $T_{(b)} = W/2$, $T_{(c)} = W/3$.

7. (a) $P(\theta) = \rho g R(1-\cos\theta) + \rho\frac{(R\omega\operatorname{sen}\theta)^2}{2}$ (b) The maximum pressure $P_{max} = \rho\frac{(g+\omega^2R)^2}{2\omega^2}$ occurs in $\theta_0 = \pi - \arccos\left(\frac{g}{\omega^2R}\right)$ if $g \leq \omega^2R$, and $P_{max} = 2\rho g R$ in $\theta_0 = \pi$ if $g > \omega^2R$.

8. The projectile moves in a highly eccentric ellipse with a semi-major axis $a = 2R$, using Kepler's three laws: $\tau = \sqrt{8R/g}\left(\frac{4\pi}{3} + \sqrt{3}\right) \approx 3\,hrs\,45\,min$.

9. It is useful to analyze the even or odd n cases, considering the motion as a parabolic shot about the surface of the incline where there is a component parallel to the surface and a perpendicular component of acceleration from 'gravity'. The condition for closed and reversible paths to form in both cases is $\alpha = \arctan\left(1/\sqrt{n+(-1)^n}\right)$.

10. Contact is lost when the relative velocity and acceleration of the block and the tip of the bar cancel. (a) $M/m = 4/3$. (b) $v = \frac{1}{4}\sqrt{3gL}$.

11. The particle's motion is analogous to that of a planetary system in a highly eccentric ellipse. From Kepler's laws $\tau = \sqrt{2m\epsilon_0}(\pi z)^{3/2}/q$.

12. If $\xi = m/M$ there will be a second collision if there is a solution to the transcendental equation $\operatorname{sen}\varphi + \xi\varphi = 0$. Approximately $\xi \leq \frac{2}{3\pi} \approx 0.21$.

13. For the equilibrium $\frac{GM}{b} = \frac{GM}{a} + \frac{\omega^2a^2}{2}$. We have as a result $f = \frac{3\omega^2}{8\pi G\rho} \approx 1.8\times10^{-3}$ (Actually this approximation is wrong by a 5/2 factor due to the assumption given for simplification. I. Newton calculated the correct result for the first time.)

14. $\mu = \frac{4h}{(4-\pi)\ell}$.

15. $h = 2R/3$ above its center.

16. $\tau = \sqrt{L/g}\ln N$.

17. Solution 1: Consider diagram I: (a) As long as time t is $OA = h = \frac{1}{2}gt^2$ the component of gravity that accelerates the beads, it tells us that the curve obeys $d(\theta) = h\cos\theta$ (this already implies that $\angle OBA = \pi/2$). This curve is a circumference of diameter h, because as seen in diagram I: $\cos\theta = y/d = d/h$, then $d^2 = hy = x^2 + y^2$. Which results in $x^2 + (y - h/2)^2 = (h/2)^2$

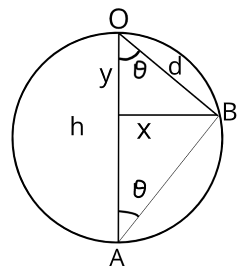


Diagram I

Solution 2: Consider diagram II: $OA = h$, $OB = h\cos\theta$ (this already implies that $\angle OBA = \pi/2$). Let C be the point in OA such that CB bisects $\angle OBA$ so $OBC = \theta$ and OBC are isosceles. ABC is also isosceles. So $OC = CA = CB = h/2$, that is why every point B is equidistant from C.

(b) In this case the curve is no longer a conic: $x^2 + (y - h/2)^2 = (h/2)^2 + ut\sqrt{x^2 + y^2}$, but a Pascal limaçon (snail), in polar coordinates: $r(\phi) = ut - \frac{1}{2}gt^2\text{sen}\phi$

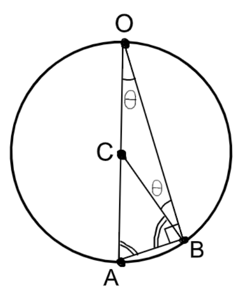


Diagram II

18. (a) $u = v(\tan 2\alpha\cos\alpha - \text{sen}\alpha)/2$ 2. (b) $0 < \alpha < \pi/4$.

19. The frictional force on the entire ring must be analyzed. (a) The path is a straight line in the same direction as $\overrightarrow{v_0}$. (b) It travels more distance when it is turning.

20. $v = v_0 e^{-\mu\pi}$.

21. It is an Archimedes spiral $r(\phi) = \frac{c}{\omega}\phi$.

22. (a) $z(r) = z_0 - \frac{c}{r^2}$.
(b) Being $\gamma = (b/a)^2$.

$$h = \frac{V}{\pi a^2}\frac{1-\gamma}{1-\gamma(1-\ln\gamma)}$$

23. $\omega = \sqrt{\frac{k}{M+\frac{m}{4}}}$.

24. (a) $v_{max} = \left(GM + \sqrt{G^2M^2 + v_0^4 b^2}\right)/v_0 b$.
(b) $r_{min} = \left(\sqrt{G^2M^2 + v_0^4 b^2} - GM\right)/v_0^2$.

25. $F \geq 2W$.

26. Of course there are many ways. A simple one is to measure the apparent change in the angular position of the chalk by closing one eye and then the other by making marks on the board. The distance is measured with the tape.

27. (a) $\rho = \frac{r_1 r_2}{r_1 - r_2}$. (b) $R = \sqrt[3]{\frac{27}{32}}r$.

28. Invariance of acceleration passing the system where the center of the circle is at rest results in $\rho = 2R$.

29. We go to the O' rest system of the radius r circumference center. In this case, acceleration $a = \frac{u^2}{\rho}$ has three contributions in this system: centripetal, centrifugal potential, and Coriolis: $a = \omega^2 r + \Omega^2(R+2r) + 2\omega r\Omega$, respectively, where Ω is the angular speed of the radius r circumference center around the radius R and ω is the angular speed of rotation of radius r circumference. The non-slip bearing condition is $\omega r = \Omega R$. $u = \omega r + \Omega(R+2r)$ is the speed of the generating point in the system O'.

Giving as a result

$$\rho = 4r\left(\frac{R+r}{R+2r}\right).$$

30. (a) $\omega_0 = \frac{24}{11}\frac{v_0}{L}$. (b) $u(t) = \frac{g}{\omega_0}\cos\omega_0 t$. (c) $v_0 = \frac{11}{12}\sqrt{gL}$.

31. (a) At the highest point, the centripetal acceleration is $u_x^2/R = g$, using conservation of energy, $u = \sqrt{5gR}$. (b) $\theta = \arctan(2) \approx 63.4°$.

32. Note that the distance is proportional to the coordinate x of the point of impact, which is at the intersection of the parabola and the line

$$x(\alpha) = \frac{v_0^2}{g}\left(\text{sen}(2\alpha) - 2\tan\beta\cos^2\alpha\right)$$

Therefore, $\cot\text{an}(2\alpha) + \tan\beta = 0$, from where we get $\alpha = \frac{\pi}{4} + \frac{\beta}{2}$.

33. $n \approx \frac{m\sqrt{g/R}}{6\pi\alpha}\left(1 - (2/3)^{3/2}\right)$.

34. (a) $h_{max} = R$. (b) $h_{max} = \frac{3}{2}R$.

35. The frequency is the inverse of the travel time difference of two consecutive wavefronts:

$$f' = f\left(1 - \frac{L - \sqrt{L^2 + (vT)^2 - 2LvT\cos\phi}}{uT}\right)^{-1}, \qquad T = \frac{1}{f} = \frac{\lambda}{c}$$

36. $F = fe^{\mu\phi}$.

37. The normal force on the table is $W + W_0$, this is also the pressure force of the liquid on the base, for a pyramid it is $3W$. Therefore, $W_0 = 2W$.

38. From the hodograph, the condition is seen so that there is only one relative clocks position. The relative speed between the clocks is $w = \sqrt{3}v$, and must also be equal to $w = 2u$ where $u = 2\pi\ell/T$ is the speed of the tip of the second hand and $T = 1\,min$. Also, $d = wT_0$ where $T_0 = 48\,hrs$

(a) $v \approx 1.2\,cm/s$, $2v \approx 2.4\,cm/s$. (b) $d \approx 3.6\,km$.

39. At the $2L/3$ distance from the end where it is held.

40. Using the result from problem 17, we have $\beta = \alpha/2$.

41. Once the cylinder begins to move with an applied force it will clear the step. $F_{min} = W\frac{\sqrt{(2R-h)h}}{R-h}$.

42. (a) First, we turn to a free-fall system O' where the velocities of each bead are constant, then consider another reference system O'' in uniform motion relative to O' such that in O'' all the beads have velocities in one direction, just of different magnitudes. Next, let's observe that when two identical beads collide their velocities are simply swapped after the collision. Giving as a result $N(n) = \frac{n(n-1)}{2}$. (b) $t = 2v/g$. For this event to occur, we need $v > 0$. (c) Listing velocities as $v_0, v_1, \cdots, v_{n-1}$, so that $v_0 = 0$ is the lowest bead velocity, one possible choice (not the only one) is $v_k = kd/t$ for $k = 0,1,\cdots,n-1$.