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Design of a Didactic Situation with Lateral Thinking to Favor the Learning of the Riemann Sum

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— Abstract—

A design proposal is presented for a teaching situation that incorporates inversion, a theoretical element of lateral thinking, and the visualization of dynamic graphics using GeoGebra. Methodologically, it is based on five moments that lead the student to construct arguments around the Riemann summation. This instrument successfully constructs a cognitive construction around the mathematical object, contrasting with the way it is presented in the books suggested by the analytical program for the Integral Calculus course in the Civil Engineering degree program at the Autonomous University of Chiapas.

Keywords:

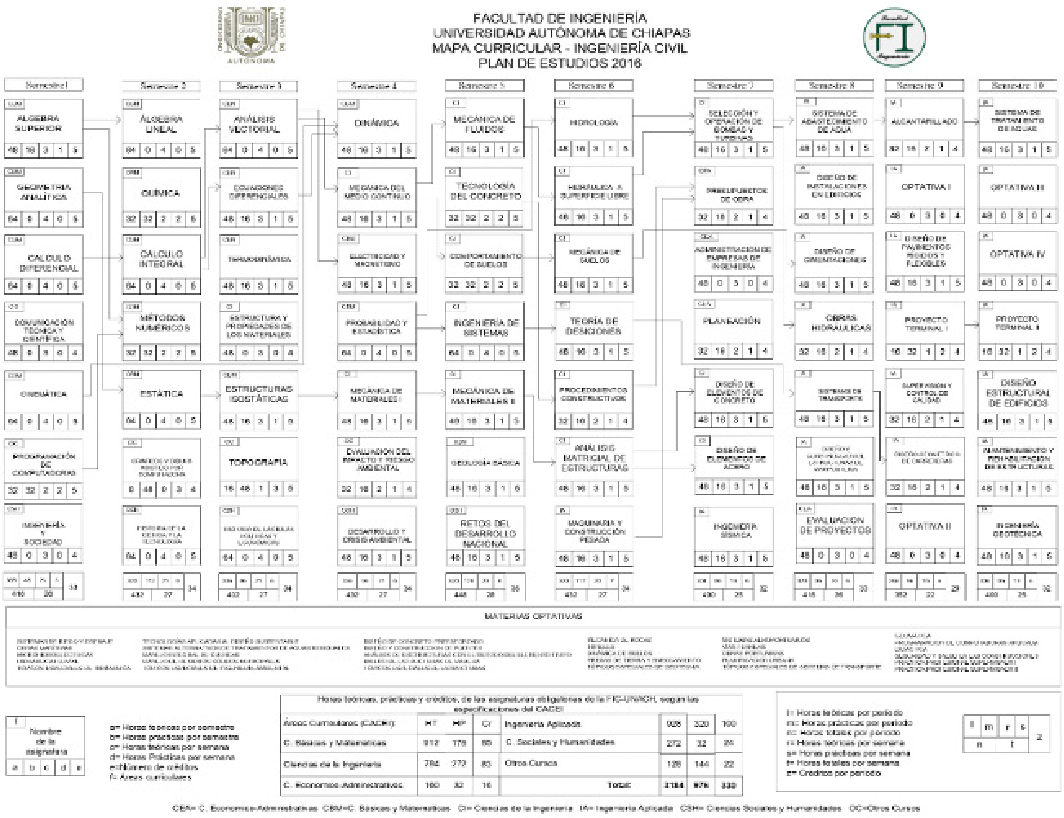
Lateral thinking, visualization in GeoGebra, summation.

The Faculty of Engineering (FE) at the Universidad Autónoma de Chiapas (UNACH) offers a Bachelor's Degree in Civil Engineering (CE), which in the first semesters teaches subjects with a mathematical content, as shown in Figure 1. The Curriculum (CURR) of the Bachelor's Degree in CE has a focus on competency-based education, which allows the student, upon graduation, to mobilize and integrate diverse knowledge and cognitive resources when faced with a new situation or problem in their workplace, so that they are able to face said situation, with constructed knowledge, propose an appropriate solution and make the most pertinent decision regarding possible courses of action, doing so effectively, and keeping in mind their ability to solve complex and open-ended problems, in different scenarios and moments.

The bachelor's degree in CE contributes to the training of professionals in Engineering through a 10-semester academic program. In order to achieve comprehensive training for the student community, competency units have been designed in various areas of training, such as life skills, basic training, and professional training.

In addition, there are optional units of competence that the student community can study in the FE, or in other educational programs of the University or at national and international Higher Education Institutions (HEI).

However, we will focus the design proposal on a mathematical content called Riemann Sum in the unit of competence called Integral Calculus (shown in the circle in Figure 1) corresponding to the second semester of the IC degree program.



Note. UNACH (2016a).

Figure 1. Curriculum map for the 2016 academic year

At UNACH, the analytical programs of each competence unit have been designed in such a way that they contribute to the development of competence attributes, which are linked to the graduate profile: knowledge, skills, attitudes and values. The curriculum and program of the bachelor's degree in CE responds to the needs and problems of today's society, as well as to other emerging issues in the field of CE application in various fields.

The current 2016 CURR includes the Integral Calculus competency unit, which aims to develop students' mathematical thinking based on modeling phenomena of variation in different contexts specific to CE, so that the student community can infer relationships and results of Calculus through a variety of real contexts. Along with analyzing and reasoning, using concepts and procedures specific to Calculus, adequately arguing decision-making and solution strategies when solving problems. Finally, to be able to effectively communicate the solutions they construct.

As part of the contents of Integral Calculus, there is the sub competence called *Definite Integral*, which is the first in the analytical program. In its content are the sequences and series, area and defined integral (properties and their respective calculation). All this structure is focused on calculating areas under the graphs of continuous functions and on the "x" axis. The purpose of this content is to understand what a sequence is in order to then build series of expressions or figures and, based on these series, construct an area under a curve (to later incorporate into a function) and above a reference (to later call it the "x" axis in a reference system). The sub competence organizes the contents in a sequential manner to reach the defined integral, which uses the series to determine the area by evaluating the anti-derivative with the initial and final values of the interval.

PERFORMANCE CRITERIA (EXPECTED LEARNING OUTCOMES)	CONTENTS
• Calculate areas under graphs, areas between graphics, and find the definite integral of different functions by applying the Riemann Sum.	**DEFINITE INTEGRAL** Sequences and series. Area. Definite Integral. Properties of the definite integral. Calculation of definite integrals.

Note. UNACH (2016b, p.151).

Figure 2. Content of the sub competence (unit) of Integral Calculus

On the other hand, in the case of *Area* content, a mathematical object called the Riemann sum is incorporated, which a calculus book defines as shown in Figure 3.

Let *f* be defined on the closed interval $[a,b]$, and let Δ be a partition of $[a,b]$ given by

$$a = x_0 < x_1 < x_2 < \cdot \cdot \cdot < x_{n-1} < x_n = b$$

where Δx_i is the width of the *i*-th subinterval. If *c1* is any point in the *i*-th subinterval. If c_1 is *any* point in the i-th subinterval $[x_{i-1}, x_i]$ then the sum

$$\sum_{i=1}^{n} f(c_i)\,\Delta x_i, \qquad x_{i-1} \leq c_i \leq x_i$$

is named a Riemann sum of f for the partition

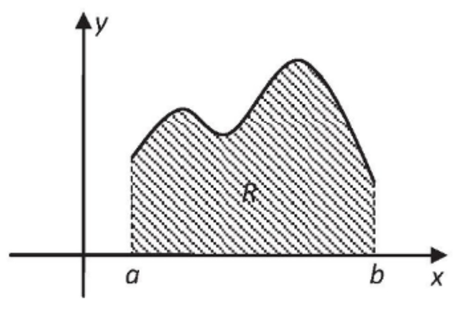
Note. Larson & Edwards (2010, p. 272).

Figure 3. Excerpt from a Calculus book showing the definition of Riemann summation

The above definition implicitly involves calculating the area of a rectangle (which would be considered below the curve), where Δ_{xi} would be the base and the value $f(c_i)$ would be its height. According to a reference from the CURR 2016, Integral Calculus analytical program, Stewart (2010, pp. 343-344), the definition shown in Figure 3 is used in another for the process of calculating the area under the curve. This other definition establishes the definite integral as a limit with a tendency to infinity for the Riemann sum.

$$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_i^n f(x_i^*)\Delta x$$

In fact, Acosta (2012) mentions that in the texts used in teaching practice, and in the teaching of calculus, the predominant approach is to introduce the concept of definite integral of a function $y = f(x)$, which is continuous and non-negative in a closed interval $[a,b]$, in close relation to the problem of determining the area of the corresponding curvilinear trapezoid, as seen in Figure 4.



Note. Acosta (2012, p. 343)

Figure 4. The definite integral of the function y = f(x) , continuous and non-negative on [a,b], is numerically equal to the measure of the area of the region R

However, this entire approach obscures the relationship between mathematical thinking that can emerge in the modeling of variation phenomena in different contexts specific to CE. Where is the inference of relations and results from the Calculus through a variety of real contexts? The student goes so far as to state that everything he was taught in the sub/competency is nothing more than to justify the use of an algorithm for the definite integral. And that is enough to use the power of series expressions that correspond to certain functions, obtain the antiderivative and get the result of the value of the area under the curve when performing the evaluation. All this leads to commenting that UNACH's FE program prioritizes the application of an algorithmic procedure without contextualizing Riemann's Sum.

In this regard, Granera (2019) points out that "in mathematics, there is little visualization and contextualization of the properties of the concepts; as well as little cognitive linkage between their graphic-visual and analytical-algorithmic aspects "(p. 5).

Regarding the point made about prioritizing the teaching of Riemann sum in the use of an algorithm for definite integrals, it agrees with the following.

> Salinas and Alanís (2009) mention that the traditional teaching of Calculus encourages teachers to focus the evaluation on the ability of students to apply algorithms and algebraic processes in the resolution of exercises. (Granera, 2019, p. 4)

It should be noted that teaching must be a conscious and intentional act on the part of the teacher, who aims to achieve learning through a series of actions that encourage the student to build their knowledge from the modeling of variation phenomena in different contexts specific to CE. On this matter, the design proposal presented in this document incorporated an element of lateral thinking into the investment. Now that, instead of presenting the analytical expression, as Calculus books do, students are asked to construct an approximation of area in a regular figure, then in an irregular figure, to finally approximate an area in a curve obtained experimentally in a context related to CE, as it is in the mechanical resistance tests on concrete masonry units (CMUs).

It is considered that investment is incorporated since in all the interaction at no time is the student presented with an analytical structure to which he or she resorts to apply algorithms and algebraic processes. Everything is based on each individual's own area approximations when interacting with the design proposal. Each closing moment in the activities suggests at the end that an analytical expression is obtained from the interaction with the parties that make up the activity.

It was stated that the investment is also implicit in the activity involving construction and mechanical testing of CMUs. Since requesting the area under the experimental stress-strain curve without providing an analytical expression for the function will cause students to resort to approximation strategies for that region. This contextualized surface alludes to the energy absorption capacity of the sample before it collapses. (Beer et al., 2010, pp. 670/671).

On the other hand, incorporating the use of GeoGebra to construct approximations of areas allows students to establish a relationship between different representation records around Riemann sum.

> GeoGebra has some representation records, such as the algebraic view, the CAS symbolic calculation view, and two-and three-dimensional graphic views, among others. These representations optimize time and allow a variety of behaviors of the object under study which will be difficult to obtain through the graphic representation with pencil and paper. (Vergara,2022, p. 2)

It was also considered that implementing GeoGebra will have an impact on dynamic visualization in the approximation of the area under the curve, since it is a software that has a degree of incidence in the teaching-learning of mathematical objects belonging to Integral Calculus. In this regard, according to Laderas et al (2023):

> The use of GeoGebra had a significant impact on the teaching and learning of Differential and Integral Calculus in higher education students. GeoGebra proved to be an effective tool both in general mathematics teaching and in specific Calculus teaching. (p. 374)

With this initial problem that was built for the Riemann sum, the aim was to develop representation records or models in GeoGebra, both geometric and analytical and numerical, in which an approximation of the area under the curve in an interval $[a,b]$ is guaranteed. As well as contextualization in CE applications, from the modeling of a stress-strain curve an approach is made to the area below the experimental curve from zero strain to the moment of rupture. The design proposal has not yet been applied to a group of students, and it is planned that this will occur in the season August-December 2025 at the FE of UNACH.

PURPOSE OF THE DESIGN PROPOSAL

The purpose of the design proposal as a *medium* was to promote learning about Riemann sums, using lateral thinking in a proposal that incorporates technology and problem-based learning. The medium is understood in the sense that Sadovsky (2005) interprets it:

> The concept of medium therefore includes both an initial mathematical problem that the subject faces, and a set of relationships, essentially mathematical as well, that change as the subject produces knowledge in the course of the situation, thereby transforming the reality with which they interact. (p. 20)

It was considered that students will develop the competences mentioned in the analytical program for the subject of Integral Calculus. The following competencies are highlighted:

- Use logical, formal mathematical, iconic, verbal and non-verbal languages to understand, interpret and express ideas and theories.
- Manage information and communication technologies as a tool for learning and collaborative work that allow their constructive participation in society.
- Use logical, critical, creative, and proactive thinking to analyze natural and social phenomena, enabling you to make relevant de-

cisions within your sphere of influence with social responsibility. (UNACH, 2016b, pp. 149-150)

Expected learning outcomes

Students will identify the composition of an area under the curve using geometric shapes such as rectangles. Said composition is an approximation of the area between the curve “f(x)” and the axis “x”.

Background knowledge

Areas of shapes (rectangles, squares, triangles, and circles), positioning, reference plane.

Who will we be working with?

For this intervention project, our subjects of study are young people between the ages of 18 and 20, from various locations in the state of Chiapas. In the second semester of the bachelor's degree in CE, it would be applied in the first sub-competency of the unit of competence called “Integral Calculus”, as marked by the analytical program for that subject.

Research question

From all the above, a research question emerged: How does a didactic sequence based on the incorporation of lateral thinking and visualization in GeoGebra favor the learning about Riemann sums in the second semester of the bachelor's degree in CE?

GENERAL OBJECTIVE

Design a teaching sequence that incorporates lateral thinking into learning about Riemann sums, incorporating GeoGebra and problem-based learning.

Specific objectives

- It is proposed to incorporate investment in the design of the didactic sequence, the approach that I have called regular-regular, which consists in that a regular area can be constituted approximately with infinite regular areas (rectangles).
- The investment in the design of the teaching sequence incorporates the approach known as irregular-regular, which consists of an

irregular area being made up of approximately infinite regular areas (triangles, rectangles, squares, circles, or others).

- Activities are proposed that incorporate the project-based learning methodology for the construction of hollow concrete blocks and the modeling of the unitary stress-strain curve.

CONCEPTUAL FRAMEWORK

The initial problem proposal or problem situation is considered to have two theoretical components: lateral thinking, on the one hand, and the visualization of graphics with the incorporation of technology in mathematics teaching, on the other.

Lateral thinking

One strategy for generating reasoning based on mathematics is lateral thinking. This concept is in contrast to the so-called vertical thinking, traditional cause-effect. Its foundations consist of taking an approach that “moves away from taking things for granted” and provocation. On the other hand, lateral thinking, "also called divergent thinking, considered by many authors as synonymous with creative thinking, which involves risk and adventure, seeks different solutions or goals in each individual, own and original" (López, 2010 in Muñoz, 2013, p. 269).

To apply it, there are different techniques (in the case of problems), among which one of them is called *Inversion*, which consists of inverting the meaning of the problem and trying to turn it into exactly the opposite. That is, an attempt is made to "rotate" it. The idea is to come up with something new that contributes to the solution of the original problem (De Luca, 2012).

We consider it relevant to invert the way the Riemann sum is displayed in a calculus course. Typically, the finished formula is presented as discussed in the introduction to this paper and applied to a standard example that develops the formula established by the textbook used by the teacher.

Incorporating technology

Using technology in the math classroom allows students to look for creative ways to solve problems, since they can focus the discussion on the meaning of mathematical ideas involved in procedures and results, because digital tools can perform mathematical calculations and procedures (Santos, 2015, p. 149).

This is where the role of the teacher comes in, given these premises, making use of lateral thinking with the incorporation of technology can help students look for creative ways to solve problems in class and not repetitive exercises. This coincides with Brousseau's proposal (1988, in Sadovsky, 2005):

> The teacher's job is therefore to present the student with a learning situation in which they can produce their knowledge as a personal response to a question and apply or modify it in response to the demands of the environment rather than the teacher's wishes. (p.28)

Since teachers can search for and find tools such as GeoGebra. That allows the design proposal for the Riemann sum a visualization of the effects that can occur between the different representation records of this mathematical object. And observe in real time the effects of the variation of the elements from one record to another.

For all these reasons, it was considered that the implementation of the technology, for the design proposal of the Riemann sum, is something feasible.

Display

It was considered that by incorporating the technology through the GeoGebra software, it allows the real-time manipulation of geometric objects, a factor that will favor the visualization and contextualization of the properties of the Riemann sum. In this regard, Granera (2019) pointed out the following:

> The findings of research show that conceptual and applied learning are scarce. Mainly in mathematics, there is little visualization and contextualization of the properties of the concepts; as well as little cognitive linkage of graphic-visual and analytical-algorithmic aspects of them. (p. 5)

On the other hand, the *Humans-with-Media* theory emphasized the role of technology in the reorganization of mathematical knowledge, considering it closely linked to *visualization*, a cognitive process that supports representation, generalization, transformation, documentation, reflection, and communication based on visual information (Hershkowitz et al., 1990; & Torregrosa, 2002, quoted in Díaz-Urdaneta & Prieto, 2016).

Diaz-Urdaneta and Prieto (2016) mentioned that in the simulation with GeoGebra, *visualization* is conceived as a cognitive process through which a selected phenomenon (natural or scientific) is represented, using mathematical ideas (in different registers) that are expanded or reorganized during the development of the activity.

We conclude this section by pointing out that we consider that these theoretical components, lateral thinking and visualization with the incorporation of technology, complement each other in an appropriate way in the design of our problem situation.

RESULTS

The result is a tool that facilitates learning about Riemann sums, which proposes a construction of the mathematical object, incorporating inversion, a theoretical approach to lateral thinking, and the visualization of dynamic graphs with GeoGebra.

METHODOLOGY OF THE PROBLEM SITUATION

The proposal is a sequence of instructions (*medium*) for students, which is designed in five stages in a start-development-end format, as proposed by Díaz (2013).

Initially, the proposal is to work with an inscribed rectangle, a circumscribed rectangle, or a combination of both characteristics, using a dynamic program for teaching and learning mathematics such as GeoGebra. In a second stage, the aim is to continue using the technology (software) and emphasis is placed on determining the appropriate number of rectangles to calculate a proposed area. In a third stage, a project is used that reflects the transition from an irregular to a regular figure. In a fourth stage, which concludes stages one, two, and three, which responds to a paradigm known as the modern conception of calculus, Moreno and Ríos (2006) noted the following about this model:

> This concept refers to learning as the construction of meaning, whereby students build knowledge based on their cultural background and guidance from the teacher. The teacher is no longer seen as a transmitter of knowledge, but as another participant in the learning process who, together with the student, constructs knowledge. This means that the teacher's activity is aimed at promoting the organization, interpretation, and understanding of the information so that the student decides what and how to learn.
>
> From this perspective, mathematical knowledge is not considered something finite, but rather knowledge in the process of creation, supported by pedagogical practices such as those promoted in the modern conception, which places conceptual structures that expand and strengthen throughout life above the storage of concepts. Thus, lectures are not sufficient; rather, scenarios must be created in which students participate in the development of their own learning. (pp. 33-34)

In a fifth moment, the entire intervention would be completed by incorporating project-based learning, which could lead to the teacher becoming another participant in the learning process and working together with the student to build mathematical knowledge.

Moments 1 and 2 consider the construction of a regular figure, which is a semicircle, using regular figures such as rectangles. Moment 3 considers the construction of an irregular figure, such as Romanesco's fern leaf; with regular geometric figures such as triangles, rectangles, squares, or circles. Moment 4 considers closing these two forms of construct. Moment 5 would conclude the entire intervention with the students. It should be noted that the teaching sequence obtained was the result of validation by a group of five teacher-researchers, who were presented with the Moment-based methodology and the proposed teaching sequence. They provided their comments, and what is shown in the results section is the final version.

Moment 1

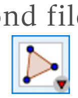
Discussion on the implementation of the shape of a rectangle that can be used to determine another area (that of the tunnel), under the assumption that we did not know that the tunnel is the outline of a circle. Lateral thinking was used by reversing the way of looking at the Riemann sum as textbooks do when presenting the constructed figure. They were provided with a GeoGebra file where, with the help of sliders, they could manipulate the number of rectangles appearing under the tunnel, as well as the intersection point of the top of the rectangle with the unknown $f(x)$.

Moment 2

After discussing the position of the rectangle in relation to the curve to be used in Moment 1, in this Moment 2 and in a second file in GeoGebra (the file will only contain the semicircle), students were able to come up with proposals regarding the number of rectangles that best suited the circular tunnel proposed in Moment 1. The following query was posed to them.

Triggering question to the student for Moment 2.

Of the shapes presented in Moment 1, what would be the best position for the rectangle to set up the tunnel area? How many rectangles would you suggest to get close to the tunnel area?

Use the second file provided in GeoGebra and use the Polygon command to draw  the rectangles inscribed in the tunnel.

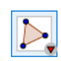
Moment 3

The aim was for students to use a regular figure to characterize an irregular figure. An irregular figure such as a fern leaf or Romanesco broccoli was presented to the students, as shown in Figure 5. The proposal was taken up again from Lima (2020), since a fractal is characterized by being a geometric object with an irregular structure that repeats itself at different scales. This property, known as self-similarity, according to this author, implies that each part of the fractal, when enlarged, shows the same shape as the entire object.

Note. Istockphoto (2025).

Figure 5. Image of Romanesco broccoli

Triggering question to the student for Moment 3.

As presented at the beginning of the session, use the  Polygon command to draw the triangles inscribed or circumscribed around the cauliflower. How many triangles would you suggest to approximate the area of the Romanesco?

Moment 4

The conclusion of moments one, two and three raises two triggering questions, which are as follows.

If the number of geometric shapes (triangles, rectangles, etc.) were very large, greater than one million geometric shapes, how close would the sum of these areas be to the area of the tunnel or the Romanesco (depending on the stage of the teaching sequence)? Explain your answer.

____

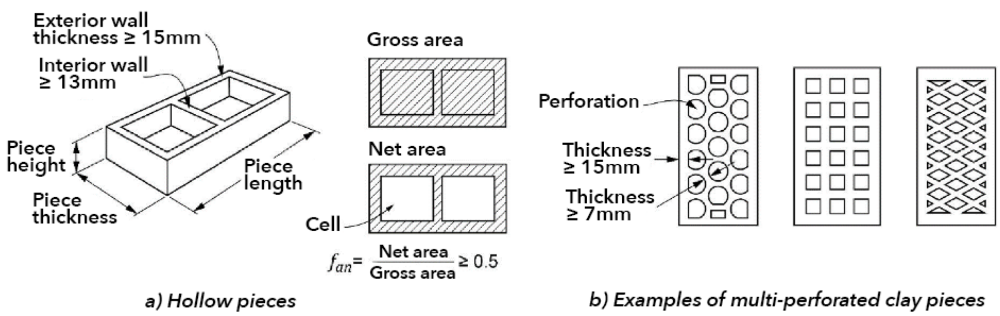
____.

What expression would you use to calculate the area of the enclosed figure using the areas of the figure or figures you proposed?

Moment 5

They proposed the construction of Hollow Concrete Block (HCB) teams, which were subjected to loading in the FE materials laboratory. The objective of this stage was to construct an HCB that must have a certain area of material that can withstand the load. As stated in the Mexican Standards ([NTC], Gavilán, 2018), hollow blocks, in their most unfavorable cross section, is a net area of at least 75 percent of the gross area; in addition, the thickness of their exterior walls is no less than 15 mm, as shown in Figure 6.

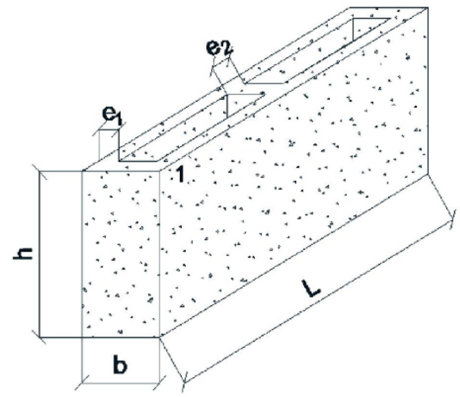
According to NTC-2018 specifications, HCB pieces with two to four cells must have a minimum interior wall thickness of 13 mm. For multi-perforated HCB pieces, whose perforations are of the same dimensions and evenly distributed, the interior walls must have a minimum thickness of 7 mm. Multi-perforated parts are understood to be those with more than seven perforations or alveoli.



Note. Gavilán (2018, p.17).

Figure 6. Specifications for hollow blocks and multi-perforated blocks

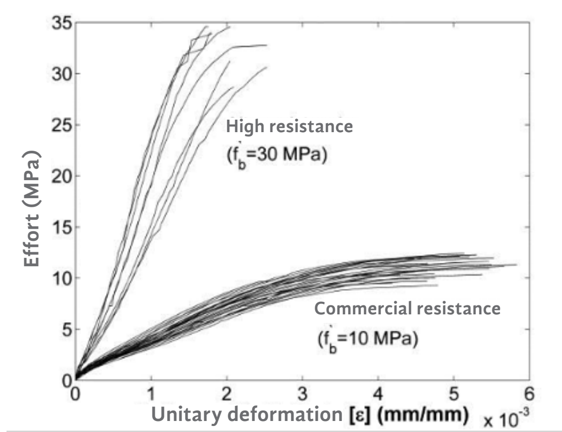
To construct a hollow concrete block (HCB), geometric characteristics are required, as indicated by Ruiz ands Godínez (2022): e1 is the thickness of the walls in the longitudinal direction; e2 is the thickness of the walls in the transverse direction, length (L), height (h), and width (b). As shown in Figure 7.



Note. Ruiz & Godínez, (2022, p. 68).

Figure 7. Relevant geometric characteristics of an HCB

Therefore, to relate the object of Riemann sum to mechanical analysis in HCB's, a continuous analysis could be performed on the mechanical tests of the students' HCBs so that they can obtain experimental stress-strain graphs of the HCBs. In García et al. (2013), concrete block samples were subjected to loads at a rate of 1 kN/s until failure. From the load versus axial deformation history of each sample, the stress-strain curves for each of the blocks tested were obtained, as shown in Figure 8.



Note. García et al. (2013, p. 79).

Figure 8. Stress-strain curves of the blocks tested at different strengths

Based on these curves, students can be asked to calculate the area under the stress-strain curve, which is interpreted as the **tenacity** of the material (Yépez, 2014). According to García et al. (2013), the experimental stress-strain curves exhibited two behaviors: Part of the curves behaved linearly (up to approximately 30% of their maximum resistance), while the next part behaved non-linearly (up to diagonal shear failure). Therefore, incorporating Riemann sum to find the area under the curve would be applicable to both the linear and nonlinear parts. Considering that the curve's function is not known analytically (unlike in Calculus textbooks, which do provide the analytical expression for the function), because it comes from an experimental process.

This Moment 5 is based on Project-based learning, in which FE students have the project of designing their HCB, with a specific number of alveoli and controlled aggregates. Following the guidelines of standard NMX-C-036-ONNCCE-2013 (2013), they will obtain the experimental stress-strain curve by measuring the displacement caused by the universal machine using an Arduino and displacement transducers (LVDTs). From this curve, it is possible to obtain the **tenacity** by approximating the area under the stress-strain curve.

A set of tasks based on solving questions or problems (challenges) is established through a process of research or creation by the student community, which works relatively autonomously and with a high level of involvement and cooperation. At the end, the process culminates in a final product presented to the others (dissemination).

Session 1

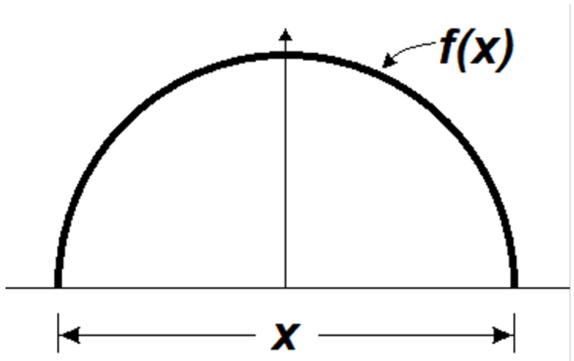
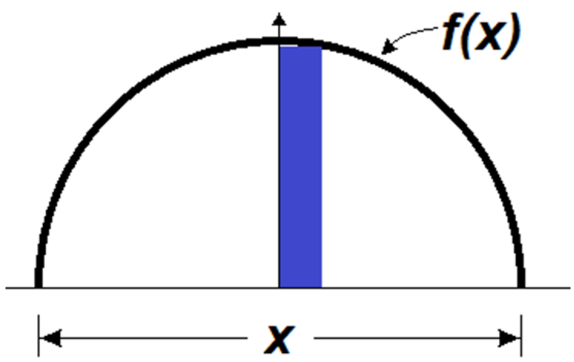
Learning objective: Students are expected to identify an ideal way to configure a rectangular area to understand another area that corresponds to a regular figure. They are provided with a file in GeoGebra so that they can propose regular figures below an f(x) without an analytical expression.

In the initial stage, students are presented with a problem in which they are required to establish an area under a curve f(x), a function that lacks an analytical representation. The teaching objective is to generate a brainstorm session with students' comments by proposing a blue rectangular figure below the semicircle and how the area of the semicircle could be covered with that rectangle or several rectangles.

Table 1

<table>
<tr><th>Stage</th><th>Activity</th><th>Time</th></tr>
<tr><td>Begi-
nning</td><td>The problem of determining the area of a tunnel using rectangular areas is presented. With an indeterminate number of rectangles (regular shape) inside a tunnel (regular shape).

Introduction to the problem

The area of a tunnel (a semicircle) must be determined by approximation. To do this, we propose constructing rectangular figures inside it, as shown in the following image.

How could the area of the semicircle be covered with that rectangle?</td><td>10 min.</td></tr>
</table>

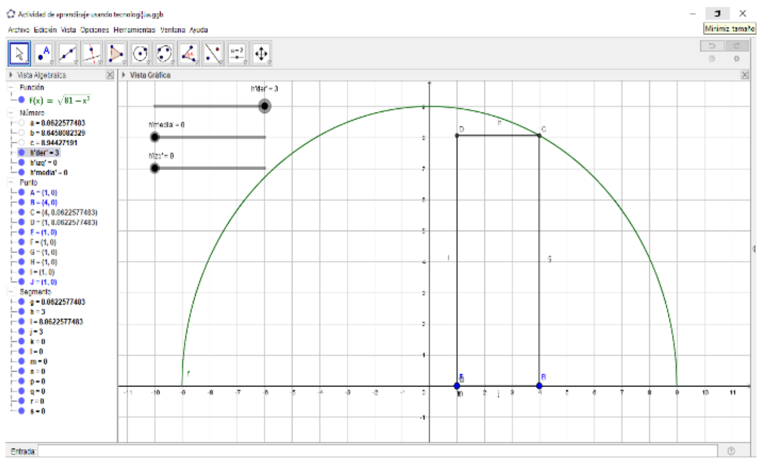
In the development stage of Session 1, the educational objective is for students to construct a series of inscribed or circumscribed rectangles with bases of equal size in the file provided by the teacher. Rectangles with different bases can also be proposed. Students are expected to grasp the idea of dynamism in the construction of rectangles and that, as their number increases, they can approximate the area under the proposed curve.

Table 2

<table>
<tr><th>Stage</th><th>Activity</th><th>Time</th></tr>
<tr><td>Deve-
lopment</td><td>Form teams of three people to work on the problem situation in Moments 1 and 2 presented in the methodology section of this document

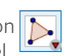
Moment 1

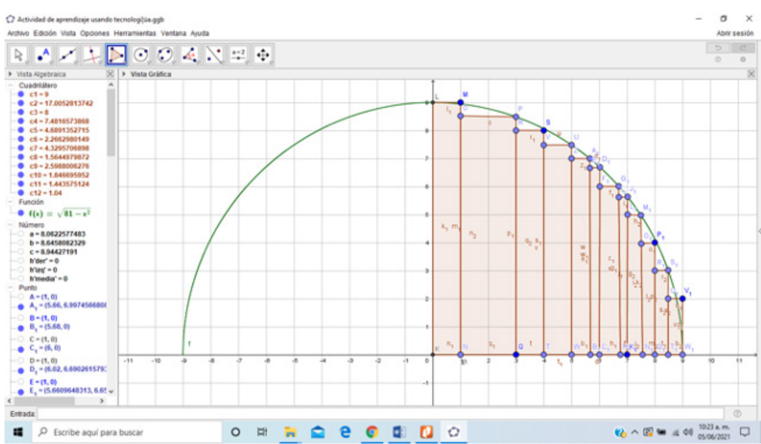
Students are provided with a file previously created in GeoGebra and instructed to: Manipulate the sliders shown on the screen one at a time. Discuss what you observe with your classmates:



Momento2

What would be the best position for the rectangle to configure the area under the tunnel you just manipulated (referring to the first GeoGebra file provided)? How many rectangles would you suggest to approximate the area of the tunnel?

Use the second GeoGebra file provided and use the Polygon  command to draw the rectangles inscribed or circumscribed around the tunnel.



You will select a team representative who will present the work done on the problem situation to the class.</td><td>80 min.</td></tr>
</table>

At the end of Session 1, the educational objective is to obtain feedback from students' answers. They should observe that by increasing the number of rectangles, they can better approximate the area of the semicircle. They are expected to construct an analytical expression for the geometric series they have proposed.

Table 3

Stage	Activity	Time
Closure	Agree on the best option that answers the question in Methodological **Moment 4**. According to your rectangular area proposal, how close is it to the tunnel area? Compare your answer with that of your classmates. If the number of rectangular shapes were very large, greater than one million rectangles, how close would the sum of these areas be to the area of the tunnel? Explain your answer. What expression would you use to calculate the area of the requested curve in a given interval using the rectangles you constructed?	25 min.

Session 2

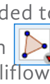
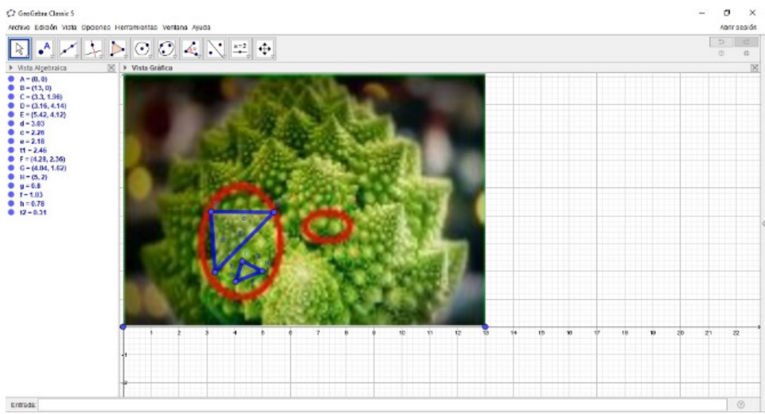
Learning Objective: Students are expected to apply a specific number of triangular areas or any other area that can be visualized in the transition from an irregular area to a regular area. In the initial stage, the educational objective is for the students to identify the self-similarity that exists in the fractal (Lima, 2020) and comment that the part enclosed in the red circle resembles the entire cauliflower. The aim is to generate a brainstorming session with the students' comments by proposing a triangular shape for the part enclosed in the circle and how the entire cauliflower area could be covered with that triangle or several triangles according to the suggestions.

Table 4

Stage	Activity	Time
Beginning	Present an irregular figure such as a fern leaf or Romanesco broccoli to the students (**Moment 3** of the methodology) Look at the circled part in the following image of a Romanesco cauliflower Does the circled red part look like the whole cauliflower?  The area of the entire cauliflower shown in the image must be determined by approximation. To do this, it is proposed to construct a triangle for the part enclosed in the circle. How could the area of the cauliflower be covered with that triangle?	15 min.

In the development stage of Session 2, the educational objective is for the students to apply a specific number of triangular areas in the transition from an irregular area to a regular one. In the initial stage, students are presented with an irregular shape and asked to determine whether a smaller area resembles the complete irregular shape. The aim is to generate a brainstorming session with students' comments when proposing a smaller area composition.

Table 5

Stage	Activity	Time
Deve-lopment	Returning the work done in the initial stage. Use the file provided to help you. Use the image of the Romanesco in GeoGebra and use the Polygon  command to draw the triangles inscribed or circumscribed around the cauliflower. Does it draw a single triangle that covers the entire surface of the cauliflower? Do you consider that the area of this single triangle is equal to the requested surface area? Try drawing more triangles to approximate the area. How many triangles would you suggest to approximate the area of the cauliflower? 	80 min.

At the end of Session 2, feedback from the students' responses is expected. They should observe that by increasing the number of triangles, they can better approximate the total area of the cauliflower. They are expected to construct an analytical expression for the geometric series they have proposed.

Table 6

Stage	Activity	Time
Closure	Socialization of your proposals (**Moment 4** of the methodology). According to the triangular area proposal, how close is it to the surface of the cau-liflower? Compare your answer with that of your classmates. If the number of triangular shapes were very large, greater than one million triangles, how close would the sum of these areas be to the surface area of the cauliflower? Explain your answer. What expression would you use to calculate the area of the surface requested in a given number of triangles that you constructed?	25 min.

Session 3

In the case of project-based learning (Moment 5), a series of activities are considered for the construction of the HCB and the modeling of the stress-strain curve.

Start of session 3

Research on how to develop an HCB and the standards it must meet. Research on how to develop a BHC and the standards it must meet. As well as the dissemination of the research findings. This coincides with the proposal by Vázquez (2021), who points out that conducting research allows students to discover new ideas, explain their opinions in a reasoned manner, apply acquired theories to practical problems, and discover new and more effective paths for their own educational process.

Development of session 3

Construction of HCB prototypes and testing in the materials mechanics laboratory to determine the experimental stress-strain curve. At this stage, students are asked to plan and organize the information obtained at the beginning of Session 3. After planning and organizing the information, they must synthesize the information that will be useful in the construction and design of the HCBs. They are also expected to identify the information that is missing for the HCB trials. This stage is based on what Aragay and Martínez (2020) mentioned about searching for and synthesizing information, as well as developing the final product.

> Information search and synthesis: In this phase, students gradually become aware of what they know and what they still need to learn. New knowledge is synthesized and linked to the needs of the project.
>
> Preparation of the final product: At this point, with the newly acquired knowledge, students are ready to respond to the challenge: the development of their final product. (p.16)

The challenge is to determine the tenacity of student-built HCBs.

Closure of session 3

Presentation of the HCB to the student community together with the results of the mechanical tests. It is considered that at this stage students will be able to pour all their experience into carrying out the BHC development project and it represents for them a much more relevant way of learning than presenting knowledge that has already been completed.

> The work that students do is much more meaningful when it is not aimed at the exam or the grade awarded by the teacher. The experiences that have developed this way of working show that when students present their work

> to a real audience, they are much more concerned about its quality. (Aragay & Martínez, 2020, p.16)

This last activity concludes Moment 5 designed in the Project-Based learning methodology.

CONCLUSIONS

The process of considering the elements that constitute a design proposal is something that has been reflected throughout the document. It is considered relevant to mention the intentions behind each part of the activities, since in many studies the sequences are presented as if they were created from scratch. A beginning, a development and a conclusion are incorporated for each of the activities. The proposal is to apply the design proposal before presenting the mathematical object, as is done in calculus textbooks.

The proposal seeks a deeper, more meaningful and contextualized learning of Riemann summation, using inversion and dynamic visualization to actively engage students in knowledge construction. It aims to break with traditional and abstract presentation, promoting understanding and mathematical reasoning in second-semester CE students at the FE of UNACH.

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